

17.  $\cos 2\theta + 2\cos^2 \frac{\theta}{2} = 1$

$\downarrow$   
 $2\left(\pm\sqrt{\frac{\cos\theta+1}{2}}\right)^2 = 2\left(\frac{\cos\theta+1}{2}\right)$   
 $2\cos^2 \theta - 1 + \cos\theta - 1 = 1$   
 $\quad \quad \quad -1 \quad -1$

$2\cos^2 \theta + \cos\theta - 1 = 0 \Rightarrow \underbrace{2\cos^2 \theta + 2\cos\theta - \cos\theta - 1}$   
 $\quad \quad \quad \underbrace{2\cos\theta(\cos\theta+1) - (\cos\theta+1)} = (\cos\theta+1)(2\cos\theta-1) = 0$   
 $\quad \quad \quad \underbrace{2 \cdot (-1) = -2} \quad \quad \quad \underbrace{2+1}$

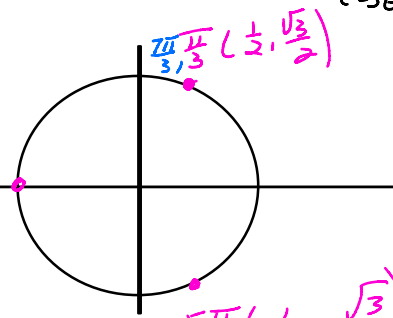
$\cos\theta+1=0$  or  $2\cos\theta-1=0$   
 $\cos\theta=-1$  or  $\cos\theta=\frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{3\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{9\pi}{3}$   
 $\quad \quad \quad \underbrace{+\frac{2\pi}{3}} \quad \underbrace{+\frac{2\pi}{3}} \quad \underbrace{+\frac{2\pi}{3}} \quad \theta = a + d(n-1)$

$\theta = \frac{\pi}{3} + \frac{2\pi}{3}(n-1)$

$\frac{\pi}{3} + \frac{2\pi}{3}n - \frac{2\pi}{3} = \frac{2\pi}{3}n - \frac{\pi}{3} = \theta$



$\frac{\pi}{3}, \frac{5\pi}{3}$

18.  $\sec^2 \theta - 2 \tan \theta = 0$

$\tan^2 \theta + 1 - 2 \tan \theta = 0$

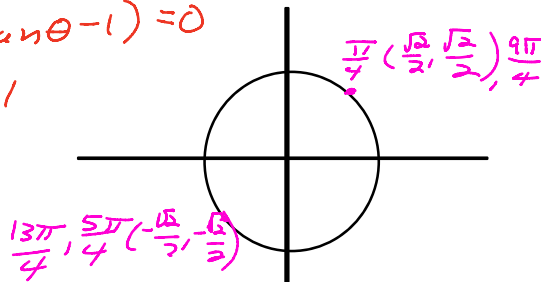
$\tan^2 \theta - 2 \tan \theta + 1 = 0 \Rightarrow \tan^2 \theta - \tan \theta - \tan \theta + 1$   
 $\tan \theta (\tan \theta - 1) - 1 (\tan \theta - 1)$

$1 \cdot 1 = 1$   
 $\quad \quad \quad \wedge$   
 $1 + 1 = 2$   
 $-1 + -1 = -2$

$(\tan\theta-1)(\tan\theta-1) = 0$   
 $\tan\theta = 1$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4} \Rightarrow \theta = \frac{\pi}{4} + \frac{4\pi}{4}(n-1)$

$\theta = \frac{\pi}{4} + \frac{4\pi}{4}n - \frac{4\pi}{4}$   
 $\theta = n\pi - \frac{3\pi}{4}$



$$16. \quad 2 \cos^2 \theta + 2 \cos 2\theta = 1$$

$$2 \cos^2 \theta - 1$$

$$2 \cos^2 \theta + 2(2 \cos^2 \theta - 1) = 1$$

$$2 \cos^2 \theta + 4 \cos^2 \theta - 2 = 1$$

+2 +2

$$6 \cos^2 \theta = 3$$

$$\sqrt{\cos^2 \theta} = \pm \frac{1}{2} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

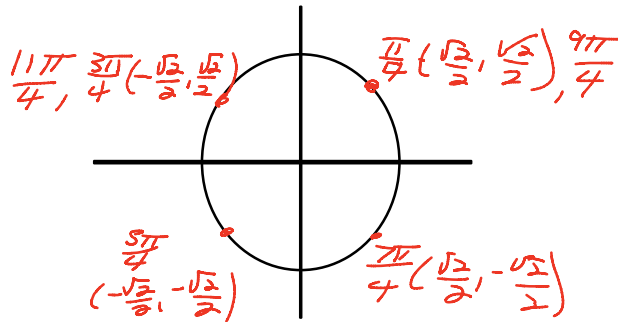
$$\cos \theta = \pm \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4} \Rightarrow \theta = \frac{\pi}{4} + \frac{2\pi}{4}(n-1)$$

$$\frac{2\pi}{4} \quad \frac{2\pi}{4} \quad \frac{2\pi}{4} \quad \frac{2\pi}{4} \quad \frac{2\pi}{4}$$

$$\frac{\pi}{4} + \frac{2\pi}{4} \cdot n - \frac{2\pi}{4}$$

$$\theta = \frac{\pi}{2}n - \frac{\pi}{4}$$



$$20. \quad \cos 2\theta \cos \theta + \sin 2\theta \sin \theta = 1$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

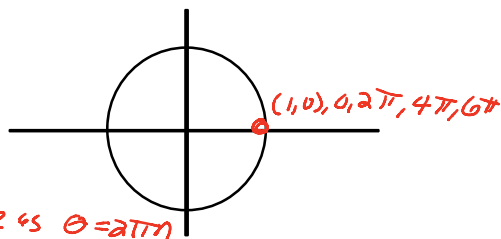
$$\cos(2\theta - \theta) = 1$$

$$\cos \theta = 1$$

$$\theta = 0, 2\pi, 4\pi, 6\pi \Rightarrow \theta = 0 + 2\pi(n-1)$$

$\underbrace{2\pi \quad 2\pi \quad 2\pi}$

$$\theta = 2\pi n - 2\pi \Rightarrow \text{same as } \theta = 2\pi n$$



$$19. \cos 2\theta - \cos \theta = 0$$

$$\downarrow \quad \quad \quad \downarrow$$

$$2\cos^2\theta - 1 - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0$$

$$\underbrace{2(-1) = -2}$$

$$\underbrace{-2 + -1 = -1}$$

$$2 + -1 = 1$$

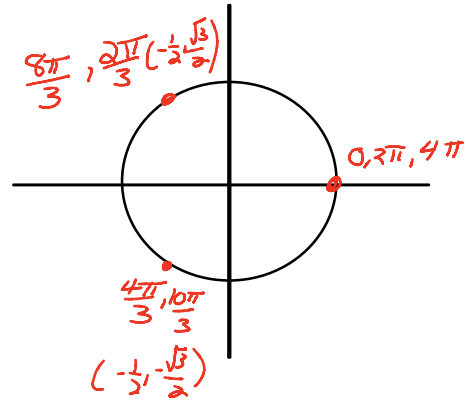
$$2\cos^2\theta - 2\cos\theta + 1\cos\theta - 1 = 0$$

$$2\cos\theta(\cos\theta - 1) + 1(\cos\theta - 1) = 0$$

$$(\cos\theta - 1)(2\cos\theta + 1) = 0$$

$$\cos\theta - 1 = 0 \quad \text{or} \quad 2\cos\theta + 1 = 0$$

$$\cos\theta = 1 \qquad \qquad \cos\theta = -\frac{1}{2}$$



$$\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$$

$$\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{6\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$$

$$\underbrace{\frac{2\pi}{3}} \quad \underbrace{\frac{2\pi}{3}}$$

$$\theta = 0 + \frac{2\pi}{3}(n-1)$$

$$\theta = \frac{2\pi}{3}n - \frac{2\pi}{3} \quad \text{or} \quad \frac{2\pi}{3}n$$

$$\cos 5\theta + \cos 3\theta = 0$$

$$\cos(4\theta + \theta) + \cos(4\theta - \theta)$$

$$\cos 4\theta \cos \theta - \sin 4\theta \sin \theta + \cos 4\theta \cos \theta + \sin 4\theta \sin \theta$$

$$2\cos 4\theta \cos \theta = 0$$

$$\cos 4\theta = 0 \text{ or } \cos \theta = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$4\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2}, \frac{17\pi}{2}, \frac{19\pi}{2}$$

4 times around

$$\frac{4\theta}{4} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}$$

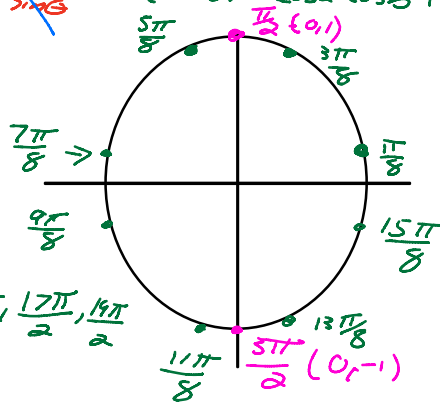
$$\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$$

Find The average of  $5\theta$  and  $3\theta$

$$\frac{5\theta + 3\theta}{2} = \frac{8\theta}{2} = 4\theta$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$



$$\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{4\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{12\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$$

$\frac{2\pi}{8}, \frac{1\pi}{8} \in$  Not The Same

create 2 generalizations

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}, \frac{17\pi}{8}$$

$$\theta = \frac{\pi}{2} + \frac{2\pi}{2}(n-1) = \frac{2\pi}{2}n - \frac{2\pi}{2} + \frac{\pi}{2}$$

$$\theta = \frac{\pi}{8} + \frac{2\pi}{8}(n-1)$$

$$\theta = \frac{\pi}{8} + \frac{2\pi}{8} \cdot n - \frac{2\pi}{8}$$

$$\frac{2\pi}{2}n - \frac{\pi}{2} = \frac{\pi}{2}(2n-1) \text{ or } \theta = \frac{2\pi}{8}n - \frac{\pi}{8} = \frac{\pi}{8}(2n-1)$$

$$\Theta = \frac{4\pi}{3} \text{ and } \frac{5\pi}{3}$$

$$11. \sqrt{3} \csc^2 \theta + 2 \csc \theta = 0$$

$$\csc \theta (\sqrt{3} \csc \theta + 2) = 0$$

$$\csc \theta = 0 \text{ or } \sqrt{3} \csc \theta + 2 = 0$$

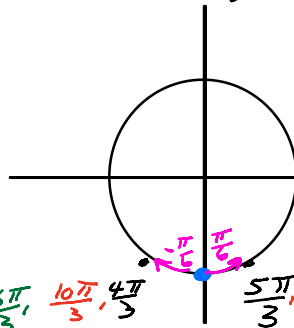
$$\cancel{\csc \theta} = 0 \text{ nope}$$

$$\csc \theta = -\frac{2}{\sqrt{3}}$$

$$\frac{1}{\sin \theta} = -\frac{2}{\sqrt{3}}$$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

generalization Find middle



$$\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \frac{15\pi}{2}$$

$$+ \frac{4\pi}{2}, \frac{8\pi}{2}, \frac{12\pi}{2}$$

$$\Theta = \frac{3\pi}{2} + \frac{4\pi}{2}(n-1)$$

$$\frac{3\pi}{2} + \frac{4\pi n}{2} - \frac{4\pi}{2}$$

$$\Theta = \frac{4\pi n - \pi}{2}$$

$$\Theta = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}, \frac{16\pi}{3}$$

$$+ \frac{1\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{3}, \frac{5\pi}{3}$$

$$\Theta = \frac{\pi}{2}(4n-1) \pm \frac{\pi}{6}$$